# Unsteady Heat and Mass Transfer from a Spheroid

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*The unsteady heat-transfer processes from oblate or prolate spheroids, at the limit of very small Peclet numbers is examined. A perturbation technique for the temperature and the geometry of the particle is used to obtain the rates of heat and mass transfer, first in the Laplace and then in the time domain. A solution to the problem is obtained, including the $\epsilon^2$ contribution ($\epsilon$ is the eccentricity). The solution reveals the existence of several history terms, which are analogous to the history terms of the creeping flow equation of motion. One of these terms is solely due to the eccentricity of the spheroid. This is an indication that the shape of the particle is a factor of the existence and form of history terms. In addition, an exact expression for the steady-state heat transfer from a spheroid is obtained using a convenient transformation of the heat-transfer integral.*

## Introduction

The subject of the steady-state heat transfer from a sphere has been well examined. There are many analytical and numerical solutions pertaining to this problem, which have been incorporated in standard textbooks and monographs, such as those by Carslaw and Jaeger (1947), Clift et al. (1978), Ozisik (1982), Leal (1991). The steady-state solutions are for both the conduction and the convection problem (Acrivos and Taylor, 1962; Brenner, 1963; Batchelor, 1979; Acrivos, 1981) at moderate Peclet numbers. However, the subject of the unsteady heat transfer has not been as widely studied. The unsteady heat transfer from particles is usually treated as a quasi-steady problem. Michaelides and Feng (1994) have recently obtained the analytical form of the unsteady energy equation for a sphere at zero Peclet numbers (*Pe*). They showed that the unsteady equation of a sphere in a conducting fluid is analogous to the creeping flow equation of motion, and discovered the existence of a history term in the energy equation. This term is analogous to the history term, which is present in the equation of motion of a sphere and is sometimes referred to as "the Basset force" or "the Basset term" (Vogir and Michaelides, 1994).

Regarding the equation of motion of a nonspherical particle, Lawrence and Weinbaum (1988) solved the equation of motion of a spheroid of revolution at the limit of zero Reynolds number and discovered "a new memory term," which stems solely from the eccentricity of the spheroid. They have shown that the history term in the equation of motion of a sphere, at the limit of zero Reynolds number (*Re*) (creeping flow conditions) is a special case of a family of such terms. These terms depend on the shape of the particle (among other variables).

Experimental evidence and everyday experience suggest that several types of initially spherical particles may be deformed to assume the shape of a spheroid in shear or uniform flows. The shape of droplets and bubbles, which move in fluids of different density slightly deviates from a sphere. This deviation is more pronounced when the Bond number is high. Spheroidal shapes of the dispersed phase have been observed in diverse applications with deformable particles, such as bubble and droplet flows in vertical or horizontal tubes, bubble and droplet flows in porous media, droplet flows in stagnant gases, as well as blood cells carried in capillaries.

In this article we present the unsteady energy transfer equation for a spheroid. We follow a method similar to the one used by Lawrence and Weinbaum (1988) for the equation of motion and use a regular perturbation method to derive the rate of heat transfer from a spheroid of small eccentricity. The final solution includes several history terms. One of them is contributed solely by the eccentricity of the spheroidal particle. The latter only appears in nonspherical particles and is analogous to the "new memory term" obtained by Lawrence and Weinbaum (1988). Finally, the steady-state solution for a spheroid is also obtained by two methods: (a) by taking the limit of the unsteady process in the Laplace domain, and (b) by utilizing a new method, which transforms the surface integral, around a particle, to an expression evaluated at infinity.

Although the following analysis applies to both the heat and the mass-transfer problem, in order to avoid circumlocution, the discussion will be limited to the heat-transfer case only. It must be emphasized, however, that all the derived equations have their analogues in the mass-transfer problem, provided that the initial and boundary conditions are the same.

## Formulation of the Problem—Governing Equations

We consider the unsteady heat transfer of a rigid spheroid inside a viscous and conducting fluid. The spheroid may have a relative velocity with respect to the fluid. When the relative velocity is small enough for the Peclet number to be much less than one, the advective part of the energy equation is much less than the conductive part and the governing equation is essentially the conduction equation:

$$\rho_f c_f \frac{\partial T_f}{\partial t} = k_f \nabla^2 T_f, \tag{1}$$

where $T_f$, $\rho_f$, $c_f$, and $k_f$ are the temperature, density, specific heat capacity, and thermal conductivity of the fluid, respectively. It must be pointed out that the assumption $Pe \ll 1$ is analogous to the creeping flow assumption in the momentum problem, $Re \ll 1$.

The fluid temperature field is decomposed into an undisturbed temperature field, $T_f^0$, and the disturbance temperature, $T_f^1$, which is solely due to the heat transfer from the spheroid. The undisturbed temperature field may be arbitrarily varying with time, but is uniform in space. It is also assumed that the temperature of the spheroid is uniform (this implies that $Bi \gg 1$). Hence, the boundary conditions imposed on the disturbance temperature field are

$$T_f^1(\boldsymbol{r},t) = T_s(t) - T_f^0(t), \tag{2a}$$

on the boundary of the spheroid, and

$$T_f^1(\boldsymbol{r},t) \to 0 \quad \text{as} \quad \boldsymbol{r} \to \infty. \tag{2b}$$

For the initial condition of the problem, it is assumed that before the commencement of the heat-transfer process, the undisturbed temperature field is uniform:

$$T_f^0(\boldsymbol{r},0) = T_f^0(0). \tag{3}$$

Regarding the geometric domain, we consider a spheroid with semiaxes $a$ and $b$ inside an infinite medium composed of the fluid. Hence, the eccentricity of the spheroid, $\epsilon$, can be defined by the expression $a = (1+\epsilon)b$, where $\epsilon$ is either positive (oblate) or negative (prolate). Hence, the equation of the surface of the spheroid is

$$\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1. \tag{4}$$

We choose as the characteristic length scale of the problem the semiaxis $b$. Hence, we define the following dimensionless variables:

$$r^* = \frac{r}{b}, \qquad T^*(r,t) = \frac{T_f^1(r,t)}{\left[T_s(0) - T_f^0(0)\right]}, \qquad t^* = t\frac{\alpha_f}{b^2}, \tag{5}$$

where $\alpha_f$ is the thermal diffusivity of the fluid. For simplicity, henceforth the asterisks will be dropped from the dimensionless variables. It should be remembered, however, that all the variables to be used are dimensionless. Hence, the governing dimensionless equations for the disturbance field and its Laplace transform are:

$$\frac{\partial T}{\partial t} - \nabla^2 T = 0 \quad \text{and} \quad (s - \nabla^2)\overline{T} = 0. \tag{6}$$

## Temperature and Domain Perturbations

It is assumed that the eccentricity of the spheroid is small but finite. Therefore, one can use the following second-order expansions in terms of the eccentricity $\epsilon$, for the surface of the spheroid in the spherical system of coordinates,

$$r = 1 + \epsilon(1-\zeta^2) + \epsilon^2 \frac{3}{2}(\zeta^4 - \zeta^2) + O(\epsilon^3),$$

where $\zeta = \cos\theta$. Similarly, we may use the following expansion for the disturbance temperature:

$$T(t,r,\zeta,\epsilon) = T_0(t,r,\zeta) + \epsilon T_1(t,r,\zeta) + \epsilon^2 T_2(t,r,\zeta) + O(\epsilon^3). \tag{8}$$

It is evident that all three components of the temperature expansion $T_0$, $T_1$, and $T_2$ must satisfy the governing equation of the problem.

For the calculations, it is advantageous to express the surface of the spheroid in terms of Legendre functions:

$$r = 1 + \epsilon f(\zeta) + \epsilon^2 g(\zeta) + O(\epsilon^3), \tag{9}$$

where the functions $f$ and $g$ are given by the following expressions:

$$f(\zeta) = \frac{2}{3}[P_0(\zeta) - P_2(\zeta)]$$

and

$$g(\zeta) = \frac{8}{35}P_4(\zeta) - \frac{2}{21}P_2(\zeta) - \frac{2}{15}P_0(\zeta). \tag{10}$$

$P_i$ denotes the Legendre polynomial of the $i$th order. The analytical form of the Legendre polynomials in Eq. (10) is as follows:

$$P_0(\zeta)=1, \qquad P_1(\zeta)=\zeta, \qquad P_2(\zeta)=\frac{1}{2}(3\zeta^2-1)$$

$$P_3(\zeta)=\frac{1}{2}(5\zeta^3-3\zeta),$$

$$P_4(\zeta)=\frac{1}{8}(35\zeta^4-30\zeta^2+3). \qquad (10a)$$

The dimensionless disturbance temperature at the surface of the spheroid is a function of time. For convenience, it will be denoted as $C(t)$, that is, $C(t)=T_s(t)-T_f^0(t)$. We perform a so-called "domain perturbation" on the surface of the spheroid around the value $r=1$ (Leal, 1992). Retaining the terms up to $O(\epsilon^2)$, we obtain the following expansion for the temperature:

$$C(t)=T_0(1,\zeta,t)+[\epsilon f(\zeta)+\epsilon^2 g(\zeta)]T_{0r}(1,\zeta t)$$
$$+\frac{1}{2}\epsilon^2[f(\zeta)]^2T_{0rr}(1,\zeta,t)+\epsilon T_1(1,\zeta,t)$$
$$+\epsilon[\epsilon f(\zeta)]T_{1r}(1,\zeta,t)+\epsilon^2T_2(1,\zeta,t)+O(\epsilon^3). \qquad (11)$$

The subscript $r$ denotes differentiation with respect to $r$. Effectively the domain perturbation technique moves the boundary conditions from $r=r_0(\zeta)$ to $r=1$, by expanding the components of $T$ in a Taylor series about $r=1$.

Collecting the terms of the different orders of $\epsilon$ in Eq. 11, we obtain the following expressions for the components of the disturbance temperature $T_0$, $T_1$, and $T_2$ at $r=1$:

$$T_0(1,\zeta,t)=C(t),$$
$$T_1(1,\zeta,t)=-f(\zeta)T_{0r}(1,\zeta,t),$$
$$T_2(1,\zeta,t)=-g(\zeta)T_{0r}(1,\zeta,t)-\frac{1}{2}[f(\zeta)]^2T_{0rr}(1,\zeta,t)$$
$$-f(\zeta)T_{1r}(1,\zeta,t). \qquad (12)$$

At infinity, $T_0=T_1=T_2=0$ since the disturbance field is local around the spheroid.

## Solution for the Unsteady Temperature Field

Since the problem is an unsteady one, its solution will be sought first in the Laplace domain, where the governing equation for the unsteady heat transfer reduces to the form: $(\nabla^2-s)\bar{T}=0$. The overline denotes the Laplace transform of a function. The solution in the Laplace domain for $\bar{T}_0$, which satisfies the imposed boundary conditions, is

$$\bar{T}_0(r,\zeta,s)=h_0^{(0)}(r,s)P_0(\zeta)$$

where

$$h_0^{(0)}(r,s)=\frac{\exp[-s^{1/2}(r-1)]}{r}\bar{C}(s). \qquad (13)$$

Substitution of Eq. 13 into the boundary conditions (Eq. 12) suggests that we must seek a solution for $\bar{T}_1$ (in the Laplace domain), which is of the form:

$$\bar{T}_1(r,\zeta,s)=h_0^{(1)}(r,s)P_0(\zeta)+h_2^{(1)}(r,s)P_2(\zeta). \qquad (14)$$

The functions $h_0^{(1)}$ and $h_2^{(1)}$ may be evaluated by substituting the expressions from Eq. 14 into the governing equation. The final expressions obtained, which satisfy the governing equation and the appropriate boundary conditions, are

$$h_0^{(1)}(r,s)=\frac{2}{3}(1+s^{1/2})\bar{C}(s)\frac{\exp[-s^{1/2}(r-1)]}{r},$$

$$h_2^{(1)}(r,s)$$
$$=-\frac{2}{3}\frac{(1+s^{1/2})(sr^2+3s^{1/2}r+3)\bar{C}(s)\exp[-s^{1/2}(r-1)]}{(s+3s^{1/2}+3)r^3}. \qquad (15)$$

The boundary conditions and the form of $\bar{T}_2(r,\zeta)$ may be similarly obtained by substituting the known solutions of $\bar{T}_0$ and $\bar{T}_1$ into Eq. 12 and the governing equation. Hence, the expression obtained for $\bar{T}_2$ is as follows:

$$\bar{T}_2(r,\zeta,s)=h_0^{(2)}(r,s)P_0(\zeta)+h_2^{(2)}(r,s)P_2(\zeta)$$
$$+h_4^{(2)}(r,s)P_4(\zeta). \qquad (16)$$

It can easily be proven that, because the functions $h_2^{(2)}(r)$ and $h_4^{(2)}(r)$ are antisymmetric, their contributions to the total rate of heat transfer are zero. Hence, the functions $h_2^{(2)}(r)$ and $h_4^{(2)}(r)$ will not be explicitly calculated. The final expression derived for $h_0^{(2)}$ is as follows:

$$h_0^{(2)}(r,s)=\frac{4}{45}\left(5s+7s^{1/2}-1-\frac{3}{s+3s^{1/2}+3}\right)$$
$$\times\bar{C}(s)\frac{\exp[-s^{1/2}(r-1)]}{r}. \qquad (16a)$$

## Heat-transfer Rate from the Spheroid—History Terms

Since the temperature field of the fluid is known, the dimensionless heat-transfer rate from the spheroid may be calculated in the Laplace domain by the following expression:

$$\bar{Q}(s)=-\oint_{S_B}\bar{q}(s)\cdot n dS=\oint_{S_B}\nabla T\cdot n\, dS. \qquad (17)$$

In order to simplify the calculations, the integral around the surface of the spheroidal particle, $S_B$, is transformed by using Gauss' theorem in the entire volume of the fluid. Considering this transformation, we observe that the contribution of the surface at infinity is zero, because of the presence of the term $\exp(-s^{1/2}(r-1))$ in the integral kernel. Utilizing the governing equation in the Laplace domain, we obtain the following expression:

$$-\oint_{S_B} \bar{q}\cdot \boldsymbol{n}\, dS = \oint_{S_B} \bar{q}\cdot(-\boldsymbol{n})\, dS + \oint_{S_\infty} \bar{q}\cdot \boldsymbol{m}\, dS$$

$$= \int_{V_f} \nabla\cdot\bar{q}\, dV = -s\int_{V_f} \bar{T}(r,\zeta)\, dV. \tag{18}$$

where $\boldsymbol{n}$ and $\boldsymbol{m}$ are the unit vectors normal to the surfaces $S_B$ and $S_\infty$, and $V_f$ is the entire fluid volume outside the particle. It must be pointed out that Eq. 18 is a very useful expression, to be used in the calculation of the rate of heat transfer from complex surfaces. The usefulness comes from the fact that, instead of calculating the surface integral of the gradient of temperature over a complex surface, one can calculate the integral of the temperature itself over the entire volume, which encloses that surface. In the case of the spheroid, the volume integral was calculated as follows:

$$\bar{Q}(s) = -2\pi s\int_{\zeta=-1}^{1}\int_{r=r_0(\zeta)}^{\infty} \bar{T}(r,\zeta) r^2\, dr\, d\zeta$$

$$= -2\pi s\int_{\zeta=-1}^{1}\int_{r=1}^{\infty} \bar{T}(r,\zeta) r^2\, dr\, d\zeta$$

$$-2\pi s\int_{\zeta=-1}^{1}\int_{r=1}^{r_0(\zeta)} \bar{T}(r,\zeta) r^2\, dr\, d\zeta \equiv -2\pi s I_1 + 2\pi s I_2. \tag{19}$$

After substituting the expressions for the disturbance temperature, $T_f^1$, and using the orthogonal properties of the Bessel functions, we obtain for the first integral $I_1$:

$$I_1 = \frac{2(1+s^{1/2})}{s}\left[h_0^{(0)}(1) + \epsilon h_0^{(1)}(1) + \epsilon^2 h_0^{(2)}(1)\right], \tag{19a}$$

and for the second integral $I_2$:

$$I_2 = \left[\frac{4}{3}\epsilon + \epsilon^2\left(\frac{5}{6} + \frac{8}{15}s^{1/2}\right)\right]\bar{C}(s). \tag{19b}$$

Hence, the final expression for the heat-transfer rate due to the disturbance field, $T_f^1$, in the Laplace domain is

$$\bar{Q}^1(s) = -4\pi\bar{C}(s)\left[\left(1+\frac{2}{3}\epsilon - \frac{1}{45}\epsilon^2\right) + \left(1+\frac{4}{3}\epsilon + \frac{2}{3}\epsilon^2\right)s^{1/2} + \frac{4s}{45(s+3s^{1/2}+3)}\epsilon^2 + O(\epsilon^2)\right]. \tag{20}$$

Equation 20 may be easily inverted from the Laplace to the time domain to yield for the rate of heat transfer, due to the disturbance field. The contribution, due to the undisturbed field $T_f^0$, is simply a term proportional to the rate of change of this temperature. When this contribution is added to $Q^1$, we obtain the total rate of heat transfer, $Q$, which is as follows:

$$Q(t) = -4\pi\left\langle\left(1+\frac{2}{3}\epsilon - \frac{1}{45}\epsilon^2\right)\left(T_s - T_f^0\right) + \left(1+\frac{4}{3}\epsilon + \frac{2}{3}\epsilon^2\right)\int_0^t \frac{\frac{d}{d\tau}\left[T_s(\tau) - T_f^0(\tau)\right]}{\sqrt{\pi(t-\tau)}}\, d\tau + \frac{1}{3}\left(1+\frac{2}{9}\epsilon + \epsilon^2\right)\frac{dT_f^0}{dt} + \epsilon^2\frac{4}{45}\sqrt{\frac{\pi}{3}}\int_0^t\left\{\frac{d}{d\tau}\left[T_s(\tau) - T_f^0(\tau)\right]\right\}G(t-\tau)\, d\tau\right\rangle, \tag{21}$$

where the function $G(t)$ is defined as follows:

$$G(t) = \mathrm{Im}\left[\sqrt{\pi\alpha}\, e^{\alpha t}\mathrm{erfc}(\sqrt{\alpha t})\right] \quad \text{and} \quad \alpha = 3e^{j(\pi/3)}. \tag{21a}$$

Since the rate of heat transfer is dimensionless, the instantaneous Nusselt number is equal to $Q(t)/2\pi$.

The first term in the righthand side of Eq. 21 is the usual conduction term. The second term is the history term, which arises from the diffusion of temperature gradients in the fluid and is similar to the one observed in the case of the sphere (Michaelides and Feng, 1994, 1995). The third term is the contribution of the undisturbed field to the energy transfer in the volume of the spheroid. This term is similar to the added mass term of the equation of motion. The fourth term is also a history term, which is equal to zero when the eccentricity is zero. Therefore, the fourth term is contributed entirely by the nonspherical shape of the particle. It must be pointed out that the second term is analogous to the usual history term (Basset force) of the equation of motion. The fourth term is analogous to the "new memory term" of the equation of motion as derived by Lawrence and Weinbaum (1988).

Equation 21 and the analogous expression for the momentum transfer, derived by Lawrence and Weinbaum (1988), are very similar in their structure. Their only differences are in the coefficients of the various terms. The presence of the "new memory term" in the momentum and energy equations of the spheroid indicates that the typical memory term, whose kernel decays as $t^{-1/2}$, may only be a particular case, applied to a spherical particle. Other terms appear in the momentum and heat-transfer equations as a result of the complex geometry of particles. The kernels of these terms do not necessarily follow the typical $t^{-1/2}$ decay. A glance at Eq. 20 proves that, at the limit of $t\to\infty$ ($s\to 0$), the Laplace transform of the new memory term is proportional to $s$. Therefore, the new memory term decays as $t^{-2}$ at $t\to\infty$. This is much faster than the typical history term. This fact is confirmed by numerical calculations.

The transient Nusselt number of a rigid spheroid was calculated when the temperature of the ambient fluid underwent a step temperature change at $t=0$. Figure 1 shows the Nusselt number, $Nu$, vs. the dimensionless time, $t$. The eccentricity of the spheroid, $\epsilon$, is the parameter in the figure ($\epsilon = 0$, 0.1, and 0.25). It is seen that the eccentricity affects

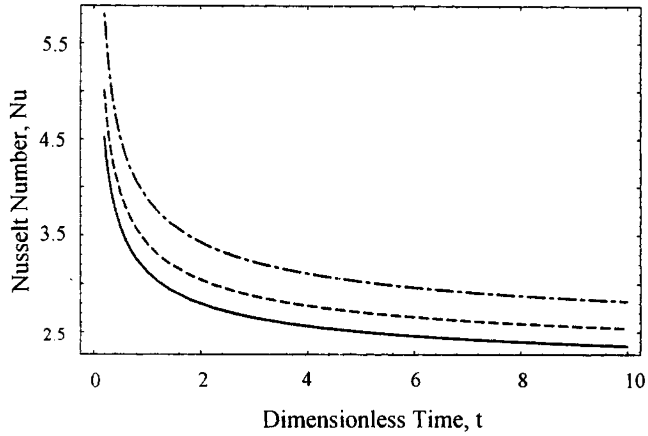


**Figure 1. Nusselt number vs. dimensionless time for a step temperature change with spheroids of $\epsilon = 0$ (——); $\epsilon = 0.1$ (----); and $\epsilon = 0.25$ (-·-·).**

substantially the rate of heat transfer at all times, and that the spheroids take longer to reach their asymptotic *Nu* values (which are 2.132 and 2.331) than the sphere (whose asymptotic *Nu* value is 2). Figure 2 depicts the contributions of the various terms of Eq. 21 to the calculated value of the Nusselt number for $\epsilon = 0.25$. The contribution of the pseudo-steady-state term is constant, equal to 2.331, and is depicted in the upper-left quadrant. The contribution of the normal memory term is shown in the upper-right quadrant. The contribution of the term, which is analogous to the added mass, is simply an impulse at $t = 0$, with total area equal to 0.777 and is not shown in the figure. The lower two quadrants depict the contribution of the new memory term (left) and the total of all contributions (right). It is observed that the contribution of the new memory term to the total Nusselt number is very small in comparison to the others and, in this case, it can be neglected. In addition, it decays faster than the other terms, as predicted by the asymptotic analysis. The contribution of the normal memory term is significant (of the same order as that of the pseudo-steady-state term) and its decay is very slow.

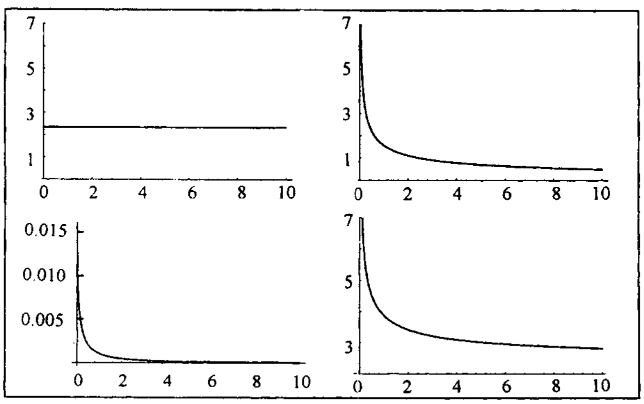


**Figure 2. Contributions of the various terms of Eq. 21 to the Nusselt number for a step temperature change and $\epsilon = 0.25$.**

Pseudo-steady-state solution in upper left; typical memory term in upper right; new memory term in lower left; total of all contributions in lower right.

## Steady-state Heat Transfer from a Spheroid

The dimensionless steady-state heat-transfer rate from the spheroid may be directly obtained from Eq. 20 by utilizing the properties of the Laplace transforms. The inversion of the resulting expression in the time domain, including the $O(\epsilon^2)$ term, is

$$Q = -4\pi\left(1 + \frac{2}{3}\epsilon - \frac{1}{45}\epsilon^2\right)\left(T_s - T_f^0\right). \qquad (22)$$

Another method has been developed for the calculation of the exact solution for the steady-state heat transfer from a particle of arbitrary shape. We will briefly describe this method, because it entails a useful technique of calculating heat-transfer integrals over complex surfaces.

In the elliptical coordinate system $(\xi, \eta, \phi)$, which is the natural system of coordinates for this problem, the steady-state conduction equation becomes (Morse and Feshbach, 1957):

$$\nabla^2 T = \frac{1}{c^2(\lambda^2 + \zeta^2)}\left[\frac{\partial}{\partial\lambda}(\lambda^2+1)\frac{\partial}{\partial\lambda} + \frac{\partial}{\partial\zeta}(1-\zeta^2)\frac{\partial}{\partial\zeta} + \frac{\lambda^2+\zeta^2}{(\lambda^2+1)(1-\zeta^2)}\frac{\partial^2}{\partial\phi^2}\right]T = 0, \qquad (23)$$

where $\lambda = \sinh\xi$, $\zeta = \cosh\eta$, and $c^2 = a^2 - b^2$. The steady-state temperature field $T(\lambda)$ is given as follows:

$$T = \frac{\tan^{-1}\left(\frac{1}{\lambda}\right)}{\tan^{-1}\left(\frac{1}{\lambda_0}\right)}. \qquad (24)$$

Hence, the rate of heat transfer is

$$\oint_{S_B} \nabla T \cdot \boldsymbol{n}\, dS = 2\pi\int_{-1}^{1}\frac{1}{h_\lambda}\frac{\partial T}{\partial\lambda}h_\zeta h_\phi\, d\zeta, \qquad (25)$$

where $h_\lambda$, $h_\zeta$, $h_\phi$ are the appropriate length scales in the spheroidal system of coordinates.

Instead of directly calculating the integral of Eq. 25, a simpler method is introduced, which makes the calculation of the rate of heat transfer in steady-state problems easier. This method is based on the fact that at a sufficiently large distance from the particle, the temperature field $T^{(s)}$ is identical to the field generated by a point-source function, $T^{(p)}$. The intensity of the point source is equal to the total rate of heat transfer $Q$. Thus, we can write:

$$\lim_{r\to\infty}\frac{T^{(s)}(\lambda)}{T^{(p)}(\lambda)} = 1. \qquad (26)$$

It is well known that the solution to the point-source problem is

$$T^{(p)} = \frac{Q}{4\pi r}. \qquad (27)$$

Substituting Eq. 27 into Eq. 26 yields:

$$Q = 4\pi \lim_{r \to \infty} (rT^{(p)}). \tag{28}$$

Since $\lim \lambda \to \infty[\lambda \tan^{-1}(1/\lambda)] = 1$, $\lambda_0 = b/c$, and at large $r$, $r \approx c\lambda$ (Morse and Feshbach, 1957), it follows that:

$$Q = \lim_{r \to \infty} \left[ c\lambda \frac{\tan^{-1}\left(\dfrac{1}{\lambda}\right)}{\tan^{-1}\left(\dfrac{1}{\lambda_0}\right)} \right] = 4\pi b \frac{\left(\dfrac{c}{b}\right)}{\tan^{-1}\left(\dfrac{c}{b}\right)}. \tag{29}$$

Equation 29 is the exact solution of the steady-state conduction from a spheroid. Physically the method of derivation of Eq. 29 is based on the fact that, at steady state any heat transfer, which is generated in a specific domain, must diffuse or be convected far from this domain (at infinity). We think that this method is very convenient and may be applied to heat-transfer problems from objects of complex shapes.

Recalling that $a = b(1+\epsilon)$ and $c^2 = a^2 - b^2$, it follows that $\lambda_0 = (\epsilon^2 + 2\epsilon)^{-1/2}$. Hence, by using an expansion formula for the function $\tan^{-1}(1/\lambda_0)$, which includes all terms up to the order $\lambda_0^{-5}$, we are able to obtain the asymptotic solution for the steady-state heat-transfer problem as follows:

$$Q = -4\pi\left(1 + \frac{2}{3}\epsilon - \frac{1}{45}\epsilon^2\right)\left(T_s - T_f^0\right). \tag{30}$$

The last expression is identical to Eq. 22, which was derived from the asymptotic expansion of the unsteady solution.

## Conclusions

A regular perturbation analysis has been used to obtain the rate of heat and mass transfer in the unsteady heat or mass diffusion from a spheroidal particle to a fluid at zero Peclet number. It is observed that the heat-transfer rate (and by analogy, the mass-transfer rate) contains history terms, which are analogous to the terms already known to be present in the equation of motion. These terms stem from the diffusion of temperature gradients, which were generated in previous stages of the process, inside the fluid. The eccentricity of the spheroid plays an important role in the form of the equations for the rate of heat transfer and the evolution of the history terms. In addition to modifying the known history terms, the eccentricity factor contributes a new history term, whose kernel does not decay as $t^{-1/2}$. It is also observed that the steady-state heat transfer may be calculated by the use of an expression that transforms a surface integral to a line integral at infinity. This method simplifies considerably the calculation of the heat- and mass-diffusion rates from complex domains.

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## Notation

$c$ = dimension equal to $a^2 - b^2$
$h$ = temperature functions
$q$ = heat flux
$r$ = radial distance
$s$ = Laplace transform variable
$\delta$ = Dirac delta function

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